STRESSED-STRAINED STATE OF SHELLS MADE OF LAYERED COMPOSITES

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Multilayer shell theory, constructed on the basis of static or kinematic hypotheses, has acquired considerable popularity [1-5]; this is explained by the physical clarity of the approach and the relative simplicity of solving specific practical problems. However, existing approaches do not make it possible to describe a simultaneously nonuniform distribution of transverse tangential stresses through the thickness of a packet, and to provide fulfilment of the continuity condition for these stresses at the interface of layers and boundary conditions at the external surfaces of a shell.

In this work on the basis of independent kinematic [1] and static [6] hypotheses, a mathematically substantiated variant of multilayer anisotropic shell theory has been constructed, taking the factors mentioned above into consideration to an equal extent. As an example, the problem is considered of an axisymmetrically stressed-strained state for a crosswise reinforced cylindrical shell.

1. We consider a shell of thickness h composed of N elastic anisotropic layers of constant thickness $h_{(k)}$ (k = 1, 2, ..., N). As a reduction surface we select the internal boundary surface Ω which is referred to a set of curvilinear coordiantes x^i. Here and subsequently, all indices, with the exception of k and m, take values of 1 and 2. Coordinate z will be read along normal $\mathbf{n}$ to the reduction surface.

The position of an arbitrary point in an undeformed shell is determined by the radius vector $\mathbf{R} = \mathbf{r} + z\mathbf{n}$, and in a deformed shell by the radius vector $\mathbf{R}^* = \mathbf{R} + u_i^{(k)}\mathbf{r}^i + u_3^{(k)}\mathbf{n}$, where $\mathbf{r}$ is radius vector of the projection of the point on surface Ω; $\mathbf{r}_i$ and $\mathbf{r}^i$ are vectors of the main and reciprocal coordinate bases; $u_i^{(k)}$ are covariant tangential displacement vector components; $u_3^{(k)}$ is normal displacement of a point in the k-th layer.

In surfaces separating layers $z = \delta_{(m)}$ (m = 1, 2, ..., N − 1) the continuity condition should be fulfilled for transverse stress tensor components $\sigma^{i3}_{(k)}$, $\sigma^{33}_{(k)}$ and the displacement vector component

$$\sigma^{i3}_{(m)} = \sigma^{i3}_{(m+1)}, \quad \sigma^{33}_{(m)} = \sigma^{33}_{(m+1)}; \tag{1.1}$$

$$u_i^{(m)} = u_i^{(m+1)}, \quad u_3^{(m)} = u_3^{(m+1)}. \tag{1.2}$$

Boundary conditions at the internal surface of the shell(z = 0) are presented in the form

$$\sigma^{i3}_{(1)} = p_0^i, \quad \sigma^{33}_{(1)} = q_0. \tag{1.3}$$

At the external surface of the shell(z = h) boundary conditions have the form

$$\sigma^{i3}_{(N)} = p_1^i, \quad \sigma^{33}_{(N)} = q_1. \tag{1.4}$$

Subsequently we use kinematic hypotheses suggested in [1]. According to [1] the material of each layer is incompressible in the transverse direction and the tangential displacement vector components of the k-th layer of the shell are linear in relation to the normal of coordinate z:

$$u_i^{(k)} = v_i^{(k)} + (z - \delta_{(k-1)})\beta_i^{(k)}, \quad u_3^{(k)} = w. \tag{1.5}$$

Here $v_i^{(k)}$ are tangential displacements for points of the "lower" boundary surface of the k-th layer; $\beta_i^{(k)}$ are increments in tangential displacements within the k-th layer.

From continuity for displacements with transfer through the layer interface and relationship (1.5), it follows that

$$u_i^{(k)} = u_i + \sum_{m=1}^{N} \pi_{(km)}\beta_i^{(m)} + z\beta_i^{(k)}, \tag{1.6}$$

Tambov. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 157-162, September-October, 1988. Original article submitted June 26, 1987.

0021-8944/88/2905-0745$12.50

where $v_i^{(1)} = u_i$; $\pi_{(km)}$ are elements of a square matrix with size N × N for which the sum of elements of the k-th line equals zero. We determine the matrix in the form

$$\|\pi_{(km)}\| = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ h_{(1)} & -\delta_{(1)} & 0 & \dots & 0 & 0 \\ h_{(1)} & h_{(2)} & -\delta_{(2)} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & & & \\ h_{(1)} & h_{(2)} & h_{(3)} & \dots & h_{(N-1)} & -\delta_{(N-1)} \end{bmatrix}.$$

It is also noted that due to the hypothesis about the incompressibility of shell layers in the transverse direction, the second continuity condition (1.2) is fulfilled automatically.

For transverse tangential stresses we take an independent approximation [6] by generalizing it in the case of boundary conditions (1.3) and (1.4):

$$\sigma_{(k)}^{i3} = p_0^i + zh^{-1}(p_1^i - p_0^i) + f_{(0)}(z)\mu_{(0)}^i + f_{(k)}(z)\mu_{(k)}^i, \tag{1.7}$$

where $f_{(0)}(z)$, $f_{(k)}(z)$ are continuous functions in the space [0, h] satisfying the conditions

$$f_{(0)}(0) = f_{(0)}(h) = 0,\ f_{(k)}(z) = 0,\ z \in [0, \delta_{(k-1)}] \cup [\delta_{(k)}, h]. \tag{1.8}$$

No limitations are imposed below on the form of functions $f_{(0)}(z)$, $f_{(k)}(z)$, and only in solving specific problems shall we assume that they are square parabolas. It can be seen from relationships (1.7) and (1.8) that transverse tangential stresses are continuous functions of the normal coordinate everywhere in region [0, h]. In this way, at boundary surfaces z = 0 and z = h, they take prescribed values $p_0{}^i$ and p_1^i.

Thus, static hypothesis (1.7) makes it possible to describe the effect of nonuniform distribution of transverse tangential stresses through the thickness of a packet without disrupting the continuity condition (1.1) and boundary conditions (1.3) and (1.4).

2. Tangential tensor components for strains $\varepsilon_{ij}^{(k)}$ and transverse shears $\gamma_{i3}^{(k)} = 2\varepsilon_{i3}{}^{(k)}$ in the case of the simplest nonlinear variant of shell theory in a quadratic approximation are determined directly by the equation

$$\begin{gathered}2\varepsilon_{ij}^{(k)} = \mathbf{R}^*_{,i}\mathbf{R}^*_{,j} - \mathbf{R}_{,i}\mathbf{R}_{,j} = \nabla_i u_j + \nabla_j u_i - 2b_{ij}w + \theta_i\theta_j + \\ + \sum_{m=1}^{N}\pi_{(km)}\left(\nabla_i\beta_j^{(m)} + \nabla_j\beta_i^{(m)}\right) + z\Big[\nabla_i\beta_j^{(k)} + \nabla_j\beta_i^{(k)} - b_i^\alpha\nabla_j u_\alpha - b_j^\alpha\nabla_i u_\alpha + \\ + 2b_i^\alpha b_{j\alpha}w - \sum_{m=1}^{N}\pi_{(km)}\left(b_i^\alpha\nabla_j\beta_\alpha^{(m)} + b_j^\alpha\nabla_i\beta_\alpha^{(m)}\right)\Big] - z^2\left(b_i^\alpha\nabla_j\beta_\alpha^{(k)} + b_j^\alpha\nabla_i\beta_\alpha^{(k)}\right), \\ \gamma_{i3}^{(k)} = \mathbf{R}^*_{,i}\mathbf{R}^*_{,3} - \mathbf{R}_{,i}\mathbf{R}_{,3} = \beta_i^{(k)} - \theta_i, \quad \theta_i = -b_i^\alpha u_\alpha - w_{,i}.\end{gathered} \tag{2.1}$$

Here an index following a comma indicates differentiation with respect to the corresponding coordinate; ∇_i is a symbol for covariant differentiation at surface Ω; a_{ij} and b_{ij} are coefficients for the first and second quadratic forms of the reduction surface. By assuming that in relationships (2.1) $\beta_i^{(k)} = \theta_i$ and considering the properties of the matrix mentioned above, $\|\pi_{(km)}\|$, we arrive at a strain relationship of the classical Kirchhoff–Love theory [7]. If it is accepted that $\beta_i^{(k)} = \beta_i$, then we obtain a deformation relationship for shell theory of the Timoshenko type [8].

Equilibrium equations for the shell are derived from the mixed variation principle of Reissner

$$\delta U = \delta A_1^* + \delta A_2^*, \tag{2.2}$$

which opens up a natural path for reducing the three-dimensional problem of elasticity theory to a two-dimensional problem of shell theory resolving on the way the well-known contradictions contained in the original system of independent kinematic and static hypotheses (1.6) and (1.7). Here A_1^* is the work of external surface loads; A_2^* is the work internal contour forces, and variation of functional U is presented in the form

$$\delta U = \iint_\Omega \left[\sum_{k=1}^{N}\int_{\delta_{(k-1)}}^{\delta_{(k)}}\left(\sigma_{(k)}^{\alpha\beta}\delta\varepsilon_{\alpha\beta}^{(k)} + \varepsilon_{\alpha\beta}^{(k)}\delta\sigma_{(k)}^{\alpha\beta} + \sigma_{(k)}^{\alpha3}\delta\gamma_{\alpha3}^{(k)} + \gamma_{\alpha3}^{(k)}\delta\sigma_{(k)}^{\alpha3} - \delta W_{(k)}\right)\sqrt{\frac{g}{a}}\,dz\right]\sqrt{a}\,dx^1\,dx^2, \tag{2.3}$$

where g and a are discriminants of the spatial metric tensor and metric tensor of the reduction surface; $W_{(k)}$ is elastic potential of the k-th layer of the shell:

$$W_{(k)}=\frac{1}{2}\left(a^{(k)}_{\alpha\beta\omega\gamma}\sigma^{\alpha\beta}_{(k)}\sigma^{\omega\gamma}_{(k)}+c^{(k)}_{\alpha\beta}\sigma^{\alpha3}_{(k)}\sigma^{\beta3}_{(k)}\right);\quad \sqrt{g/a}=1-2Hz+Kz^2; \tag{2.4}$$

H and K are average and Gaussian curvature of surface Ω; $a^{(k)}_{\alpha\beta\omega\gamma}$, $c^{(k)}_{\alpha\beta}$ are tangential and transverse shear compliances of the k-th layer. It is noted for information that $c^{(k)}_{\alpha\beta}=4a_{\alpha3\beta3}{}^{(k)}$.

By introducing $W_{(k)}$ from (2.4) into Eq. (2.3), after a standard variation procedure we obtain an expression for variation of functional U:

$$\delta U=\iint_{\Omega}\Big\{\left(-\nabla_\alpha T^{\alpha\omega}+b^{\omega}_{\alpha}N^{\alpha}\right)\delta u_\omega+\left(-\nabla_\alpha N^{\alpha}-b_{\alpha\omega}T^{\alpha\omega}\right)\delta w+$$
$$+\sum_{k=1}^{N}\left(-\nabla_\alpha\Phi^{\alpha\omega}_{(k)}+Q^{\omega}_{(k)}\right)\delta\beta^{(k)}_{\omega}+\sum_{k=1}^{N}\int_{\delta_{(k-1)}}^{\delta_{(k)}}\left(\gamma^{(k)}_{\alpha3}-c^{(k)}_{\alpha\omega}\sigma^{\omega3}_{(k)}\right)\left[f_{(0)}(z)\,\delta\mu^{\alpha}_{(0)}+\right.$$
$$\left.+f_{(k)}(z)\,\delta\mu^{\alpha}_{(k)}\right]\sqrt{\frac{g}{a}}\,dz\Big\}\sqrt{a}\,dx^1dx^2+\oint_{\Gamma}\left[T_{\nu\nu}\delta u_\nu+T_{\nu t}\delta u_t+N_{\nu3}\delta w+\sum_{k=1}^{N}\left(\Phi^{(k)}_{\nu\nu}\delta\beta^{(k)}_{\nu}+\Phi^{(k)}_{\nu t}\delta\beta^{(k)}_{t}\right)\right]ds_t; \tag{2.5}$$

$$T^{ij}=\sum_{k=1}^{N}T^{ij}_{(k)},\quad Q^{i}=\sum_{k=1}^{N}Q^{i}_{(k)},\quad S^{ij}=\sum_{k=1}^{N}S^{ij}_{(k)}, \tag{2.6}$$
$$N^{i}=Q^{i}-S^{\alpha i}\theta_\alpha,\quad \Phi^{ij}_{(k)}=M^{ij}_{(k)}+\sum_{m=1}^{N}\pi_{(mk)}T^{ij}_{(m)},$$
$$T^{ij}_{(k)}=\int_{\delta_{(k-1)}}^{\delta_{(k)}}\left(\sigma^{ij}_{(k)}-zb^{j}_{\alpha}\sigma^{i\alpha}_{(k)}\right)\sqrt{\frac{g}{a}}\,dz,\quad S^{ij}_{(k)}=\int_{\delta_{(k-1)}}^{\delta_{(k)}}\sigma^{ij}_{(k)}\sqrt{\frac{g}{a}}\,dz,$$
$$M^{ij}_{(k)}=\int_{\delta_{(k-1)}}^{\delta_{(k)}}\left(\sigma^{ij}_{(k)}-zb^{j}_{\alpha}\sigma^{i\alpha}_{(k)}\right)z\sqrt{\frac{g}{a}}\,dz,\quad Q^{i}_{(k)}=\int_{\delta_{(k-1)}}^{\delta_{(k)}}\sigma^{i3}_{(k)}\sqrt{\frac{g}{a}}\,dz$$

($T_{\nu\nu},\ldots,\Phi^{(k)}_{\nu t}$, $u_\nu,\ldots,\beta^{(k)}_{t}$ are physical components of the corresponding tensors and vectors in coordinate system s_t, s_ν, connected with boundary contour Γ).

By calculating the variation of work for the external loads and substituting the values found δA^{*}_{1}, δA^{*}_{2} together with δU from (2.5) in variation Eq. (2.2), after simple transformations according to [8,9], we obtain an equation for shell equilibrium in specific forces and moments:

$$\nabla_\alpha T^{\alpha i}-b^{i}_{\alpha}N^{\alpha}=p^{i}_{0}-p^{i}_{*},\quad \nabla_\alpha N^{\alpha}+b_{\alpha\omega}T^{\alpha\omega}=q_0-q_*, \tag{2.7}$$
$$\nabla_\alpha\Phi^{\alpha i}_{(k)}-Q^{i}_{(k)}=-h_{(k)}p^{i}_{*}\quad (k=1,2,\ldots,N),$$
$$p^{i}_{*}=(1-2hH+Kh^2)\,p^{i}_{1},\quad q_*=(1-2hH+Kh^2)\,q_1,$$

under conditions corresponding to them:

$$T_{\nu\nu}=T^{*}_{\nu\nu}\ \text{or}\ u_\nu=u^{*}_{\nu},\quad T_{\nu t}=T^{*}_{\nu t}\ \text{or}\ u_t=u^{*}_{t},$$
$$N_{\nu3}=Q^{*}_{\nu3}\ \text{or}\ w=w^{*},$$
$$\Phi^{(k)}_{\nu\nu}=\Phi^{(k)*}_{\nu\nu}\ \text{or}\ \beta^{(k)}_{\nu}=\beta^{(k)*}_{\nu},\quad \Phi^{(k)}_{\nu t}=\Phi^{(k)*}_{\nu t}\ \text{or}\ \beta^{(k)}_{t}=\beta^{(k)*}_{t},$$

and also, taking account of (2.4), the integral relationships

$$\sum_{k=1}^{N}\int_{\delta_{(k-1)}}^{\delta_{(k)}}\left(\gamma^{(k)}_{i3}-c^{(k)}_{i\alpha}\sigma^{\alpha3}_{(k)}\right)f_{(0)}(z)\left(1-2zH+Kz^2\right)dz=0,$$
$$\int_{\delta_{(k-1)}}^{\delta_{(k)}}\left(\gamma^{(k)}_{i3}-c^{(k)}_{i\alpha}\sigma^{\alpha3}_{(k)}\right)f_{(k)}(z)\left(1-2zH+Kz^2\right)dz=0. \tag{2.8}$$

As can be seen from Eq. (2.8) elasticity relationships for transverse tangential stresses are fulfilled integrally through the thickness of the k-th layer and simultaneously throughout the thickness of the packet, and hereby a nonlinear variation of multilayer anisotropic shell theory is constructed which is not contradictory from the point of view of the mixed variation principle, taking account of the nonuniform distribution of the stress tensor component throughout the thickness of the shell.

3. We consider axisymmetrical deformation of a thin multilayer anisotropic shell of rotation. Surface Ω relates to the curvilinear orthogonal coordinates α_1 and α_2 read along the line of principal curvature. In this case relationships (2.1), (2.6)-(2.8) are considerably simplified. After neglecting terms $k_i z$ in comparison with unity, whose retention does not increase the accuracy of the final results, deformation relationships (2.1) are written in the form

$$\begin{gathered}\varepsilon_{(ij)}^{(k)} = E_{(ij)} + \sum_{m=1}^{N} \pi_{(km)} K_{(ij)}^{(m)} + z K_{(ij)}^{(k)},\\ E_{11} = \frac{1}{A_1}\frac{du_1}{d\alpha_1} + k_1 w + \frac{1}{2}\theta_{(1)}^2, \quad E_{(22)} = k_2 w - \rho u_{(1)} + \frac{1}{2}\theta_{(2)}^2,\\ 2E_{(12)} = \frac{1}{A_1}\frac{du_{(2)}}{d\alpha_1} + \rho u_{(2)} + \theta_{(1)}\theta_{(2)}, \quad K_{(11)}^{(k)} = \frac{1}{A_1}\frac{d\beta_{(1)}^{(k)}}{d\alpha_1},\\ K_{(22)}^{(k)} = -\rho\beta_{(1)}^{(k)}, \quad 2K_{(12)}^{(k)} = \frac{1}{A_1}\frac{d\beta_{(2)}^{(k)}}{d\alpha_1} + \rho\beta_{(2)}^{(k)},\\ \theta_{(1)} = k_1 u_{(1)} - \frac{1}{A_1}\frac{dw}{d\alpha_1}, \quad \theta_{(2)} = k_2 u_{(2)}, \quad \rho = -\frac{1}{A_1 A_2}\frac{dA_2}{d\alpha_1},\end{gathered} \tag{3.1}$$

where $\varepsilon_{(ij)}^{(k)}, \ldots, \theta_{(i)}$ are physical components corresponding to tensors and vectors; k_i is curvature of coordinate lines; A_i are Lamé parameters. Similar simplification should be carried out with equilibrium Eqs. (2.7), specific forces and moments (2.6), and integral elasticity relationships (2.8).

The structure of the original equations for nonlinear multilayer anisotropic shell theory is quite complex. Accurate solutions can be obtained in rare cases, and therefore we shall aim at their numerical solution in a computer. With this in mind we introduce a vector for solutions with a size of 4N + 6:

$$\begin{gathered}\mathbf{Y} = [T_{(11)}, N_{(1)}, \Phi_{(11)}^{(1)}, \ldots, \Phi_{(11)}^{(N)}, T_{(12)}, \Phi_{(12)}^{(1)}, \ldots, \Phi_{(12)}^{(N)},\\ u_{(1)}, w, \beta_{(1)}^{(1)}, \ldots, \beta_{(1)}^{(N)}, u_{(2)}, \beta_{(2)}^{(1)}, \ldots, \beta_{(2)}^{(N)}]^T.\end{gathered} \tag{3.2}$$

According to Eq. (3.2) construction of the vector for solutions makes it possible to present the resolvent equations of the problem in matrix form. The normal set of simple differential equations has the form

$$\frac{1}{A_1}\frac{d\mathbf{Y}}{d\alpha_1} = \mathbf{F}(\alpha_1, \mathbf{Y}) \tag{3.3}$$

(expressions for vector component **F** are not provided here). The first 2N + 3 equations of system (3.3) follow from equilibrium Eqs. (2.7) taking account of the equality $S_{(ij)} = T_{(ij)}$, and the other 2N + 3 equations flow directly from deformation relationships (3.1). Transverse specific forces $Q_{(i)}^{(k)}$ figuring in the right-hand parts of set (3.3) may be expressed in terms of vector components for solutions by means of integral elasticity relationships (2.8).

An algorithm for solution of the formulated problem was realized in the form of a collection of standard procedures in algorithmic language PL/1(O). All of the numerical calculations were carried out on an ES 1060 computer.

4. As an example we consider a crosswise reinforced cylindrical shell, one of whose ends is displaced by a prescribed distance u_0. The layers are packed in an antisymmetrical way with reinforcing angles $\gamma_k = (-1)^{k-1}\gamma$. The problem is realized numerically for a two-layer shell with geometric parameters h = 5 mm, $R_0 = \ell = 100$ mm (R_0 is radius, ℓ is shell length), prepared from a boron-epoxy composite. The mechanical properties of the composite are presented in

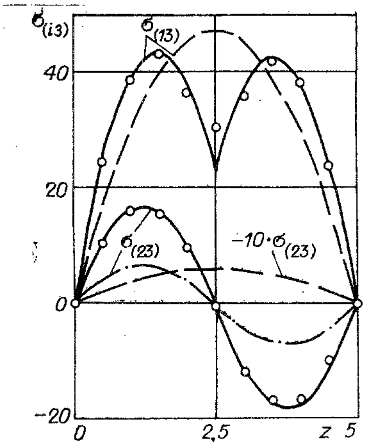

Fig. 1

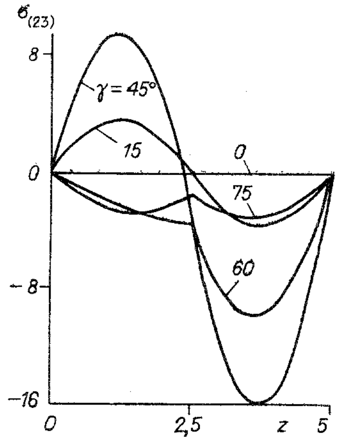

Fig. 2

[10]. It was assumed that the shell ends are rigidly embedded, and axial displacement u_0 = 1 mm.

Shown in Fig. 1 is the distribution of transverse tangential stresses through the thickness of the packet in a section of the shell located at a distance of 10 mm from the end with a reinforcement angle $\gamma = 30°$. Numerical results represented by solid lines were obtained by integrating the normal set of simple differential equations (3.3). Broken lines correspond to calculations based on Timoshenko-type shell theory [8]; broken-dotted lines are the refined Timoshenko theory [9], on whose basis static hypothesis (1.7) is also suggested, although the order of resolvent equations in this theory does not depend on the number of layers. Points are the results of solving the problem by the finite element method [11] where the shell was considered from the position of nonlinear elasticity theory. As can be seen, stresses $\sigma_{(13)}$ are distributed through the thickness of the packet by a rule close to parabolic, although at the interface of layers considerable deviation is observed from the square parabola rule. As far as stresses $\sigma_{(23)}$ are concerned, they generally have a nonparabolic distribution which is postulated in the overwhelming majority of refined theories for multilayer shells. In the problem being considered, their distribution rule is very close to sinusoidal. The order of the values of $\sigma_{(13)}$, $\sigma_{(23)}$ is the same, which points to the considerable contribution of the effect of anisotropy to the overall picture of the stressed-strained state of a crosswise reinforced shell. It is curious to note that the refined Timoshenko theory [9] characterized quite well the rule for distribution of transverse tangential stresses $\sigma_{(13)}$. The values obtained for these stresses are so close to those found by means of the theory developed here that they agree with an accuracy up to the scale of representation.

In conclusion we analyze the nature of change in transverse tangential stress $\sigma_{(23)}$ curves in relation to reinforcement angle γ. Numerical results are presented in Fig. 2. It can be seen that curves for stresses $\sigma_{(23)}$ depend markedly on reinforcement angle, and the nature of distribution for these stresses through the thickness of the shell undergoes a qualitative change.

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OPTIMIZATION OF THE STRUCTURE OF ROLLED SHELLS

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UDC 539.3

1. In [1], the direct problem of determining the stressed state of a cylindrical tube prepared by rolling a thin flexible shell is considered. The elastiplastic model for deformation of these structures is the following closed set of equations:

$$\frac{\partial \sigma_{11}^0}{\partial \lambda_1} + \frac{\partial \sigma_{12}^0}{a_2 \partial \lambda_2} + \frac{\sigma_{11}^0 - \sigma_{22}^0}{a_2} = 0, \qquad \frac{\partial \sigma_{12}^0}{\partial \lambda_1} + \frac{\partial \sigma_{22}^0}{a_2 \partial \lambda_2} + \frac{2\sigma_{12}^0}{a_2} = 0; \tag{1.1}$$

$$\frac{\partial w_1^0}{\partial \lambda_1} = \frac{1-\nu}{2\mu}\sigma_{11}^0 - \frac{\nu}{2\mu}\sigma_{22}^0, \qquad \frac{\partial w_2^0}{a_2 \partial \lambda_2} + \frac{w_1^0}{a_2} = \frac{1-\nu}{2\mu}\sigma_{22}^0 - \frac{\nu}{2\mu}\sigma_{11}^0; \tag{1.2}$$

$$\frac{\partial w_2^0}{\partial \lambda_1} + \frac{\partial w_1^0}{a_2 \partial \lambda_2} - \frac{w_2^0}{a_2} = \frac{\sigma_{12}^0}{\mu} + \Gamma(\sigma_{12}^0, \lambda_1, \lambda_2). \tag{1.3}$$

Here (λ_1, λ_2) is the orthogonal curvilinear coordinate system; line $\lambda_1 = \text{const}$ is directed along the contact of shell layers; $a_2 = \lambda_1 + \xi\lambda_2 + R_0 \cos \delta$ is Lamé parameter; $\xi = R_0 \sin \delta$; R_0 is tube internal radius; δ is slope of spiral λ_2 to circle $r = R_0$; w_i^0, σ_{ij}^0 $(i, j = 1, 2)$ are displacement vector components and the stress tensor in coordinates (λ_1, λ_2); μ is shear modulus; ν is Poisson's ratio. Set (1.1) are normal equilibrium equations in curvilinear coordinates; (1.2) are equations determining the elastic change in dimensions of an elementary volume in directions λ_1 and λ_2; (1.3) characterizes the overall shear strain of an element of the material; the first term in the right-hand part is elastic deformation of shell layers; Γ is slippage of layers over each other. This stressed-strained state depends markedly on the form of function Γ, i.e., on the conditions at the contacts between shell layers. This situation may be used for optimizing the structure as a whole.

Let the shell be intended for operation at high internal pressures when as a best performance criterion we take

$$p \to \max \tag{1.4}$$

(p is the value of internal pressure). It is noted that this criterion should be fulfilled with prescribed internal pressure, material parameters μ, ν, shell layer thickness $h = 2\pi\xi$, and fulfillment of certain inequalities guaranteeing material integrity. Thus, if Eq. (1.3) is excluded from the closed set (i.e., Γ is considered as a controlling function), then best performance condition (1.4) may be used in order to obtain equations closing set (1.1), (1.2). After solving it from Eq. (1.3), where displacements and stresses are already known, we determine function Γ, which provides fulfillment of criterion (1.4). This is the general scheme for solving the problem.

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 162-167, September-October, 1988. Original article submitted Aprkl 4, 1987.

0021-8944/88/2905-0750$12.50